Utilizing the Kelly Criterion to Select the Best Projects When Capital is Temporarily Constrained

James A. MacKay and Gary P. Citron

Rose and Associates, LLP, 7660 Woodway Dr., Ste. 590, Houston, Texas 77063

GCAGS Explore & Discover Article #00026*
http://www.gcags.org/exploreanddiscover/2016/00026_mackay_and_citron.pdf
Posted September 13, 2016.

*Article based on a full paper published in the *GCAGS Transactions* (see footnote reference below), which is available as part of the entire 2016 *GCAGS Transactions* volume via the GCAGS Bookstore at the Bureau of Economic Geology (www.beg.utexas.edu) or as an individual document via AAPG Datapages, Inc. (www.datapages.com), and delivered as an oral presentation at the 66th Annual GCAGS Convention and 63rd Annual GCSSEPM Meeting in Corpus Christi, Texas, September 18–20, 2016.

ABSTRACT

The Kelly Criterion, developed in 1956 by John Kelly at the Bell Laboratories, provides a method to allocate capital to a project with the intent of maximizing the return on the capital employed and limiting exposure to a critical shortfall in the total capital available for other projects. This shortfall can occur when projects that are funded early in the funding cycle are subjected to a run of bad luck and both the corporate success rate and value added from exploration falls significantly lower than expected. This disappointment could cause a tactical revision to the budget and diminish the pool of capital available for the remaining projects.

Even when this criterion has already been applied to balance the portfolio with the corporate risk attitude and the capital available, the budget may be subjected to a sudden reduction in the remaining funds available due to reasons beyond their control. This constraint may possibly be due to temporary cash flow shortages, another corporate division with a sudden need for capital or as we have seen in the last six months the need to pay down debt. Because the constrained budget is not a change in corporate attitude regarding money to be placed at risk, but rather a temporary economic remedy to a shortage of cash currently available, the company may prefer to reduce the budget year allocation but maintain the corporate risk attitude. To do this the company must determine which projects in the portfolio best meet the corporate objectives for maximizing long term return at an appropriate level of risk and either reduce equity or postpone some projects to meet the cash flow constraints.

This paper will suggest one method to make the required adjustments based on a linear programming model. A linear program solution is similar to a marble dropped into a tilted box. The marble will come to rest at the intersection of the two sides that form the lowest location in the box. It will not find a solution if, for instance, one side is perfectly aligned with the low point such that all the points on that edge are equally low or if there are baffles that prevent the marble from continuing to roll to the lowest point. Other more robust models such as non-linear or integer programming might find a solution in these more complex situations.

Originally published as: MacKay, J. A., and G. P. Citron, 2016, Utilizing the Kelly criterion to select the best projects when capital is temporarily constrained: Gulf Coast Association of Geological Societies Transactions, v. 66, p. 357–362.

Rose & Associates

Utilizing the Kelly Criterion to Select the Best Projects when Capital is Temporarily Constrained

James A. MacKay
Gary P. Citron

In 2015 presented the Kelly Criterion in a paper and poster session at the GCAGS convention in Houston.

The Kelly Criterion: A Review

1. The Kelly Criterion is used by the financial industry to allocate capital.
2. The Kelly Criterion balances preserving wealth and adding value.
3. MacKay (2015) modified the formulas for the oil industry and published the results in last year's GCAGS proceedings.

The Kelly criterion is designed to allocate wealth to a project.

The Kelly Criterion: A Review

4. The modified formulas can be used with a linear program to adjust the various project working interests and maximize overall return.
5. Kelly can also be modified to adjust for any corporate risk tolerance (that is, reduced amounts financial exposure available to your projects due to a variety of concerns).

Although the Kelly criterion is intended to be applied to individual projects sequentially the method can be modified to apply to a portfolio simultaneously.

Calculating The Kelly Criterion

The Kelly Criterion (K) is a simple, easy to create ratio of Expected Value (EV) to Present Value (PV)

$$K = EV / PV$$

I have modified and simplified the Kelly criterion to be the ratio of expected value over present value.

Calculating The Kelly Criterion

Kelly Criterion for an upcoming Project B:

Gain (G) = $10; Cost (C) = $3; Chance (Ps) = 50%

PV = G – C = 10 – 3 = $7

EV = (Ps x G) – C = (0.5 x 10) – 3 = $2

K = $2 / $7 = 29%

This is the 5D objectives slide

Calculating The Kelly Criterion

Kelly Criterion for an upcoming Project B:

Gain (G) = $10; Cost (C) = $3; Chance (Ps) = 50%

PV = G – C = 10 – 3 = $7

EV = (Ps x G) – C = (0.5 x 10) – 3 = $2

K = $2 / $7 = 29%

This represents the percentage of capital that should be allocated to Project B in a 'risk neutral' setting.

The result is the percent of capital to should be allocated to this project to both preserve and maximize wealth.

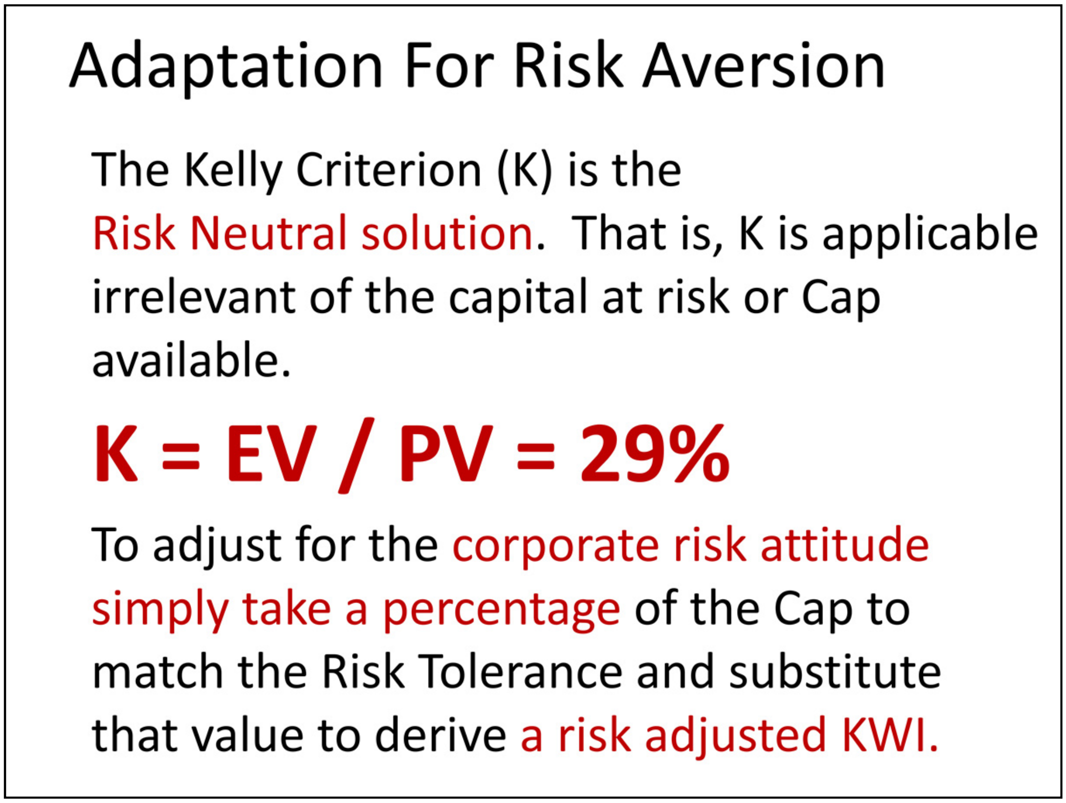
Utilizing The Kelly Criterion

Therefore 29% of whatever capital (Cap) is available should be applied to Project B (up to a maximum of 100% of the cost), resulting in the Kelly Working Interest (KWI).
So, if Cap = $5; and Cost (C) = $3

KWI = (K x Cap) / C

KWI = (0.29 x 5) / 3 = 48%

That percentage of wealth needs to be converted to a percentage of the project.



IF you are risk averse then simply use a fixed percentage of Capital to lower the risk (and lower the long term return).

Adaptation For Risk Aversion

So our previous

KWI = (K x Cap) / C

KWI = (0.29 x 5) / 3 = 48%

Is adjusted by RT = 50% of Cap = $2.5

KWIra **= (K x RT) / C**

KWIra **= (0.29 x 2.5) / 3 = 24%**

Due to the limit of 100% of cost a 50% adjustment may not always result in a 50% reduction in working interest.

Portfolio Application:

Portfolio of three (3) Projects

Parameter and Definition	Project	A	B	C	
Cost	C	$ 5.00	$ 3.00	$ 2.00	
Gain	G	$ 7.50	$ 10.00	$ 20.00	
Present Value	PV = G-C	$ 2.50	$ 7.00	$ 18.00	
Chance	Ps	100%	50%	25%	
Available Capital	Cap	$ 5.00	$ 5.00	$ 5.00	
Expected Value	EV = Ps*G-C	$ 2.50	$ 2.00	$ 3.00	
Kelly Criterion	K = EV/PV	100.0%	28.6%	16.7%	Total
Kelly Working Interest	KWI = K*Cap/C	100.0%	47.6%	41.7%	Cost
Kelly cost	Ck = KWI*C	$ 5.00	$ 1.43	$ 0.83	$ 7.26
				Budget	$ 5.00

Challenge: The Total Cost $7.26 exceeds $5.00 Budget

In this portfolio the ideal Kelly investments for the three projects results in a total cost of $7.26 million. Unfortunately the budget is only $5 million.

Portfolio Application:

Reprioritization Options

Portfolio of just A

Parameter and Definition	Project	A			
Cost	C	$ 5.00			
Gain	G	$ 7.50			
Present Value	PV = G-C	$ 2.50			
Chance	Ps	100%			
Available Capital	Cap	$ 5.00			
Expected Value	EV = Ps*G-C	$ 2.50			
Kelly Criterion	K = EV/PV	100.0%			Total
Kelly Working Interest	KWI = K*Cap/C	100.0%			Cost
Kelly cost	Ck = KWI*C	$ 5.00			$ 5.00
				Budget	$ 5.00

Tradeoff: Not diversified, little growth potential without B and C

Project A alone provides only nominal growth.

Portfolio Application:

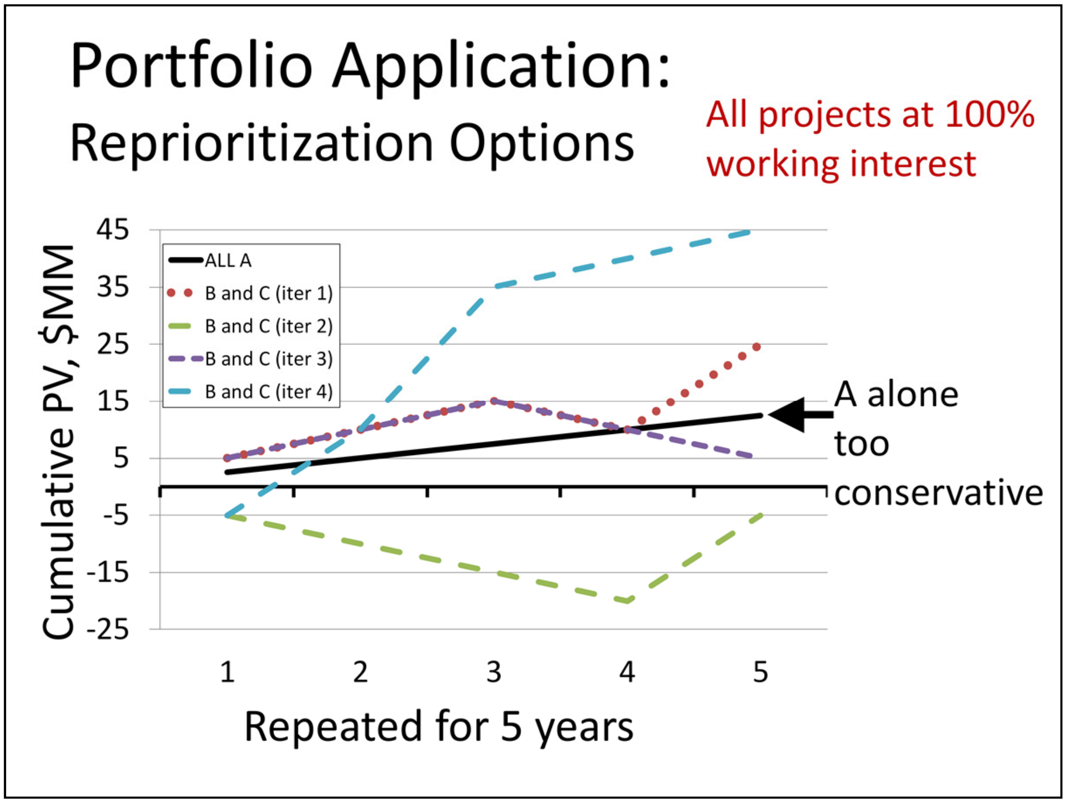
Reprioritization Options

Portfolio of just B and C

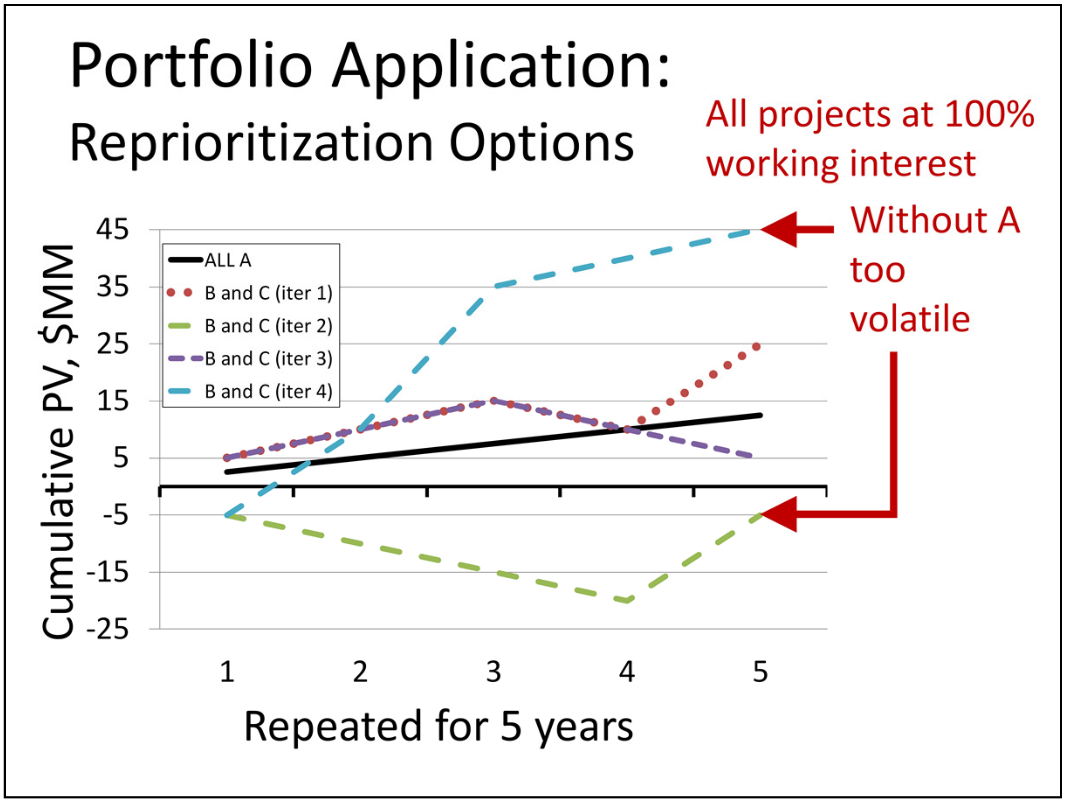
Parameter and Definition	Project		B	C	
Cost	C		$ 3.00	$ 2.00	
Gain	G		$ 10.00	$ 20.00	
Present Value	PV = G-C		$ 7.00	$ 18.00	
Chance	Ps		50%	25%	
Available Capital	Cap		$ 5.00	$ 5.00	
Expected Value	EV = Ps*G-C		$ 2.00	$ 3.00	
Kelly Criterion	K = EV/PV		28.6%	16.7%	Total
Kelly Working Interest	KWI = K*Cap/C		47.6%	41.7%	Cost
Kelly cost	Ck = KWI*C		$ 1.43	$ 0.83	$ 2.26
				Budget	$ 5.00

Tradeoff: lower chance of value gain
money sent back to treasury

Projects B and C could be funded at 100% each but combined are too volatile.



This shows the possible change in wealth is the same program was repeated for 5 years. A alone is to conservative and B and C are too volatile.



This shows the possible change in wealth is the same program was repeated for 5 years. A alone is to conservative and B and C are too volatile.

Portfolio Application:

Reprioritization with KWI Adjustment

Reduce the KWI to match budget

Parameter and Definition	Project	A	B	C	Total
Kelly Working Interest	KWI = K*Cap/C	100.0%	47.6%	41.7%	Cost
Kelly cost	Ck = KWI*C	$ 5.00	$ 1.43	$ 0.83	$ 7.26
Adjustments due to Budget less than Total Cost				Budget	$ 5.00
(AWI) Adj Working Interest	= KWI * Bud/Total Cost	68.9%	32.8%	28.7%	
Adjusted Cost		$ 3.44	$ 0.98	$ 0.57	$ 5.00

What is the adjusted long term value?

To adjust the portfolio to meet the budget constraint all three projects could be reduced to a combined total cost of $5 million.

Portfolio Application:

Optimize the Geometric Mean Value

The Kelly Criterion is based on maximizing the Geometric Mean Value (GMV) of the Portfolio

GMV = [(Cap + (PV x WI))^Ps x (Cap – (C x WI))^Pf)] – Cap

The GMV serves to balance preserving wealth and adding value

The Kelly criterion suggests a better alternative based on maximizing the geometric mean.

Portfolio Application:

Optimize the Geometric Mean Value

Using a linear program such as Solver can Maximize the portfolio GMV by adjusting the selected project working interests and constraining the budget.

The GMV serves to balance preserving wealth and adding value

A linear program is used to adjusted the working interests to maximize the geometric mean while constraining the budget.

Portfolio Application:

Optimization increases the Kelly Geometric Mean Value

`	Project	A	B	C	
Adjustments due to Budget less than Total Cost				**Budget**	**$ 5.00**
(AWI) Adjusted WI	= KWI * Bud/Total Cost	**68.9%**	**32.8%**	**28.7%**	
Adjusted Cost	= AWI * C	**$ 3.44**	**$ 0.98**	**$ 0.57**	**$ 5.00**
Kelly Value	**as per GMV**	**$ 1.72**	**$ 0.41**	**$ 0.45**	**$ 2.58**
Expected Value		**$ 1.72**	**$ 0.66**	**$ 0.86**	**$ 3.24**
Adjustments via Solver using GMV constrained to Budget					
Constrained WI		**85.3%**	**10.6%**	**21.0%**	
Constrained Kelly Cost		**$ 4.27**	**$ 0.32**	**$ 0.42**	**$ 5.00**
Kelly Value at CWI	**as per GMV**	**$ 2.13**	**$ 0.18**	**$ 0.39**	**$ 2.71**
Expected Value		**$ 2.13**	**$ 0.21**	**$ 0.63**	**$ 2.97**

From $2.58 to $2.71,

While honoring the $5.00 budget

The result maximizes the geometric mean but not the expected value.

Portfolio Application:

Optimization increases WI at A, decreases WI at B & C

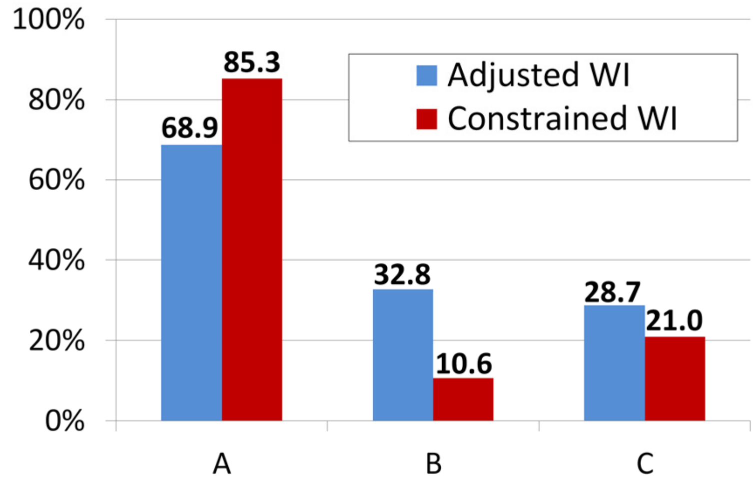
`	Project	A	B	C	
Adjustments due to Budget less than Total Cost				**Budget**	**$ 5.00**
(AWI) Adjusted WI	= KWI * Bud/Total Cost	**68.9%**	**32.8%**	**28.7%**	
Adjusted Cost	= AWI * C	**$ [illegible]4**	**$ [illegible]8**	**$ [illegible]7**	**$ 5.00**
Kelly Value	**as per GMV**	**$ [illegible]2**	**$ [illegible]1**	**$ [illegible]5**	**$ 2.58**
Expected Value		**$ [illegible]2**	**$ [illegible]**	**$ [illegible]**	**$ 3.24**
Adjustments via Solver using GMV constrained to Budget					
Constrained WI		**85.3%**	**10.6%**	**21.0%**	
Constrained Kelly Cost		**$ 4.27**	**$ 0.32**	**$ 0.42**	**$ 5.00**
Kelly Value at CWI	**as per GMV**	**$ 2.13**	**$ 0.18**	**$ 0.39**	**$ 2.71**
Expected Value		**$ 2.13**	**$ 0.21**	**$ 0.63**	**$ 2.97**

While honoring the $5.00 budget

The result maximizes the geometric mean but not the expected value.

Portfolio Application:

Optimization increases WI at A, decreases WI at B & C



While honoring the $5.00 budget

Summary and Conclusions:

1. The Kelly Criterion (K = EV/PV) is easy to understand.

   K can be used in a non-complicated, straight forward fashion to calculate the optimum working interest (KWI) for each project.

   KWI = K x Cap / C

Summary and Conclusions:

2. Optimization is readily available.

   If the total cost for all projects exceeds the budget the working interests can be further adjusted using the geometric mean values (GMV) of each project and a linear program such as solver.

Summary and Conclusions:

3. The Kelly Criterion can readily adjust to any risk tolerance levels.

   If the lower budget reflects a long term change in strategy a new corporate risk tolerance can and should be calculated.

The speaker, James MacKay, can be contacted at jamesmackay@roseassoc.com.

REFERENCES cited in the manuscript:

- Kelly, John L. Jr. (1956). "A New Interpretation of Information Rate." Bell System Technical Journal, Vol. 35, pp. 917-926.
- Hung, Jane. (2010). University of Washington, Department of Mathematics, *Betting with the Kelly Criterion*, June 2, 2010 http://www.math.washington.edu/~morrow/336_10/papers/jane.pdf (accessed January 21, 2016)
- MacKay, James A. (2015). "Comparing the Results of the Kelly Criterion and Risk Aversion: Quick Look Practical Alternatives to Portfolio Optimization." GCAGS Transactions (2015) Vol. 65, pp. 273-280.
- Poundstone, William (2005). "Fortune's Formula: the Untold Story of the Scientific Betting Systems that Beat the Casinos and Wall Street." New York. Hill and Wang, 385 pgs.